Higher Derivative Corrections to Locally Black Brane Metrics

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ABSTRACT: In this paper we generalize the construction of locally boosted black brane space time to higher derivative gravities. We consider the Gauss-Bonnet term (with coefficient α') as a toy example. We find the solution to the α' corrected Einstein equations to first order in the boundary derivative expansion. This allows us to find the α' corrections to the boundary stress tensor in the presence of the Gauss-Bonnet term in the bulk action. We therefore obtain the ratio of shear viscosity to entropy which agrees with other methods of computation in the literature.

KEYWORDS: AdS/CFT, Hydrodynamics, Higher derivative.

Contents

1. Introduction

The conjectured duality between thermal gauge theory and gravity in one higher dimensional AdS spacetime is a useful tool to extract information about strongly coupled large **N** gauge theory. For example different thermodynamic variables like free energy, entropy *etc* of large **N SU(N)** gauge theory in $(\mathbf{3}+\mathbf{1})$ dimensions in large 't Hooft coupling regime can be conjectured by calculating the thermodynamic variables of black hole in five dimensional **AdS** spacetime.

If we consider a black object with translation invariant horizon, for example black **D3** brane geometry, one can also discuss hydrodynamics - long wave length deviation (low frequency fluctuation) from thermal equilibrium. In addition to the thermodynamic quantities the black brane is also characterised by the hydrodynamic parameters like viscosity, diffusion constant *etc*. The black **D3** brane geometry with low energy fluctuations (*i.e.* with hydrodynamic behaviour) is dual to some finite temperature gauge theory plasma living on boundary with hydrodynamic fluctuations. Therefore studying the hydrodynamic properties of strongly coupled gauge theory plasma using the AdS/CFT duality is an interesting subject of current

research. The energy momentum tensor of a relativistic viscous conformal fluid is given by (up to first order in derivative expansion),

$$T_{\mu\nu} = (e+p)u_\mu u_\nu + p\eta_{\mu\nu} - 2\eta\sigma_{\mu\nu} \tag{1.1}$$

where u_μ is fluid 4-velocity with $u_\mu u^\mu = -1$, e is energy, p is pressure and η is shear viscosity coefficient. $\sigma_{\mu\nu}$ is defined in 2.13. Conformal invariance implies that $e = 3p$. The first attempt to study hydrodynamics via AdS/CFT was [1]. The authors related the shear viscosity coefficient η of strongly coupled $\mathcal{N} = 4$ gauge theory plasma in large **N** limit with the absorption cross-section of low energy gravitons by black **D3** brane. Other hydrodynamic quantities like speed of sound, diffusion constants, drag force on quarks *etc* can also be computed in the context of AdS/CFT (See [2] for review and the references therein).

If we consider the 't Hooft coupling to be very large but finite, then we have to include the string theory contributions to thermodynamic and hydrodynamic quantities, *i.e.* we need to improve the supergravity results by including the higher derivative terms in the action. The higher derivative (string theory) corrections to shear viscosity have been calculated in [4],[5],[6] [1]. There exists a "viscosity bound conjecture" [3] which states that the viscosity to entropy ratio $\frac{\eta}{s}$ has a lower bound,

$$\frac{\eta}{s} \geq \frac{1}{4\pi} \tag{1.2}$$

for all relativistic quantum field theories at finite temperature. In fact in presence of the $\mathbf{R^4}$ term which is the first higher derivative correction appears in type II string theory the viscosity entropy ratio is greater than $\frac{1}{4\pi}$. But the presence of Gauss-Bonnet term in the Lagrangian seems to violate the "viscosity bound conjecture". Interested readers are referred to follow [5], [6], [8] for detailed discussions. In all the cases the transport coefficients have been determined either by the Kubo formula (graviton absorption) or by quasi-normal mode calculation [9],[10] or using the membrane paradigm approach [11].

Recently [2] in [12] the authors have developed an elegant systematic framework to construct the nonlinear fluid dynamics, order by order in boundary derivative expansion. The five dimensional Einstein equations with a negative cosmological constant with appropriate boundary conditions can be reduced to nonlinear equations of fluid dynamics. In this small note we have generalised the construction of local black brane geometry to higher derivative gravity. We started with Gauss-Bonnet term as a toy model. We have found the solution of α' corrected Einstein equations only up to first order in boundary derivative expansion. Once we obtained the α' corrected geometry we calculate the boundary stress tensor up to first order in derivative expansion. From the expression of stress tensor one can read the α' correction to shear viscosity coefficient. We have also calculated the ratio of shear viscosity to entropy and the result agrees with existing results in literature calculated in other ways [5], [6]. Though we have found the correction to the metric up to first order in derivative

[1] See also [7].

[2] Also in [13], authors have discussed second order hydrodynamics for conformal fluid. See also [14], [15] and [16] for related discussion.

expansion, but it would be interesting to find the corrections to the metric and stress tensor up to second order in derivative expansion [27].

We proceed in the same way of [12]. The presence of Gauss-Bonnet term preserves all the symmetries of AdS_5 spacetime. We have also worked in the Eddington-Finkelstein coordinate. Our solutions are also non-singular away from $r = 0$, and specially at the location of the horizon.

The plan of the paper is following. In section 2 we will very briefly sketch the calculation frame work of [12] and in section 3 we will display our results. We finish our paper with some concluding remarks (section 4).

2. Fluid Dynamics From Gravity: The Computational Framework

In this section we will briefly sketch the working procedure of [12]. For detailed discussion readers are referred to the original paper. We will also skip the technical details in this section.

- Consider the Einstein-Hilbert action with negative cosmological constant

$$I = -\frac{1}{16\pi G_5}\int d^5x\sqrt{-g}\left(R + \frac{12}{L^2}\right) \tag{2.1}$$

  where L is the radius of AdS space.

- The equation of motions are given by [3]

$$E_{MN} = R_{MN} - \frac{1}{2}Rg_{MN} - \frac{6}{L^2}g_{MN} = 0. \tag{2.2}$$

- There exists a class of solutions to these equations of motion given by the "boosted black branes" [4],

$$ds^2 = -2u_\mu dx^\mu dr - \frac{r^2}{L^2}f(br)u_\mu u_\nu dx^\mu dx^\nu + \frac{r^2}{L^2}\left(u_\mu u_\nu + \eta_{\mu\nu}\right)dx^\mu dx^\nu \tag{2.3}$$

  with,

$$\begin{aligned} f(r) &= 1 - \frac{1}{r^4}, \\ u_v &= -\gamma \\ and\;\; u_i &= \gamma\beta_i \end{aligned} \tag{2.4}$$

  where, $\gamma = 1/\sqrt{1-\vec{\beta}^2}$.

[3] $x^M = \{v, r, \vec{x}\}$.
[4] $x^\mu = \{v, \vec{x}\}$.

- Putting the values of u_μ's the metric can also be written as,

$$\begin{aligned} ds^2 = \quad & 2\gamma dv dr - \frac{r^2}{L^2}\gamma^2 f(br) dv^2 + \frac{r^2}{L^2} dx^i dx^i \\ + & \frac{r^2}{L^2}(\gamma^2 - 1) dv^2 - 2\gamma \beta_i dx^i dr - 2\frac{r^2}{L^2}\gamma^2 (1 - f(br))\beta_i dx^i dv \\ + & \frac{r^2}{L^2}\gamma^2 (1 - f(br))\beta_i \beta_j dx^i dx^j . \end{aligned} \tag{2.5}$$

  The solution is parametrised by four constant parameters b and β_i's.

- The black brane horizon is located at $r_H = 1/b$ and the temperature of this black brane is given by,

$$T = \frac{1}{\pi b L^2}. \tag{2.6}$$

- Consider the metric 2.5 and replace the constant parameters b and β_i's by slowly varying functions $b(x^\mu)$ and $\beta_i(x^\mu)$'s of boundary coordinates x^μ

$$\begin{aligned} ds^2 = \quad & 2\gamma dv dr - \frac{r^2}{L^2}\gamma^2 f(b(x^\alpha) r) dv^2 + \frac{r^2}{L^2} dx^i dx^i \\ + & \frac{r^2}{L^2}(\gamma^2 - 1) dv^2 - 2\gamma \beta_i(x^\alpha) dx^i dr - 2\frac{r^2}{L^2}\gamma^2 (1 - f(b(x^\alpha) r))\beta_i(x^\alpha) dx^i dv \\ + & \frac{r^2}{L^2}\gamma^2 (1 - f(b(x^\alpha) r))\beta_i(x^\alpha) \beta_j(x^\alpha) dx^i dx^j . \end{aligned} \tag{2.7}$$

  We will call this metric $g^{(0)}(b(x^\alpha), \beta_i(x^\alpha))$.

- In general the metric 2.7 is not a solution to Einstein equations unless one adds some corrections to the metric and also the parameters $b(x^\alpha), \beta_i(x^\alpha)$ satisfy some set of equations, which turn out to be the equations of boundary fluid mechanics.

- Write the parameters $b(x^\alpha)$ and $\beta_i(x^\alpha)$ and the metric as a derivative expansion of the parameters. Up to first order in derivative expansion,

$$g = g^{(0)}(b(x^\alpha), \beta_i(x^\alpha)) + \epsilon g^{(1)}(b(x^\alpha), \beta_i(x^\alpha)), \tag{2.8}$$

$$b(x^\alpha) = b^{(0)}(x^\alpha) \tag{2.9}$$

  and

$$\beta_i(x^\alpha) = \beta_i^{(0)}(x^\alpha) \tag{2.10}$$

  where ϵ is a dimension less parameter whose power counts the number of (boundary)spacetime derivatives acting on the parameters. Since $b^{(1)}(x^\alpha)$ and $\beta_i^{(1)}(x^\alpha)$ do not enter in to the first order equation of motions, we have kept the expansion for b and β_i's up to leading order.

- In general one can write the metric and parameters as power series of ϵ. Then plug the metric in Einstein equations and solve the metric and the parameters order by order (in ϵ). For example in our case since we are interested up to first order, we will plug the metric in Einstein equations and solve for $g^{(1)}$ and the constraint equations imply some relations between the zeroth order parameters. We will work in a particular gauge,

$$Tr((g^{(0)})^{-1}g^{(1)}) = 0. \tag{2.11}$$

- After finding the metric with first order fluctuations one can find the boundary stress tensor (using the definition given in [17], [18]). The form of the boundary stress up to first order in derivative expansion is given by,

$$16\pi G_5 T_{\mu\nu} = T^4\pi^4 L^3 \left(4u_\mu u_\nu + \eta_{\mu\nu}\right) - 2T^3\pi^3 L^3 \sigma_{\mu\nu}, \tag{2.12}$$

where $\sigma_{\mu\nu}$ is given by,

$$\sigma_{\mu\nu} = P_\mu^\alpha P_\nu^\beta \partial_{(\alpha} u_{\beta)} - \frac{1}{3} P_{\mu\nu} \partial_\alpha u^\alpha \tag{2.13}$$

and $P_{\mu\nu} = u_\mu u_\nu + \eta_{\mu\nu}$.

3. Higher Derivative Correction to First Order Hydrodynamics

In this section we will explicitly show how one can generalise this procedure to higher derivative action. We will consider the Gauss-Bonnet action as a toy example. We will show how the first order metric and constraint relations receive α' corrections. As a result the boundary stress tensor also receives an α' correction. We have considered the Gauss-Bonnet term as a perturbation and hence our metric and stress tensor is correct up to first order in α'.

3.1 The Action

We will start with following action,

$$I = -\ \frac{1}{16\pi G_5} \int_{\mathcal{M}} d^5x \sqrt{-g} \left(R + \frac{12}{b^2}\right) - \frac{\alpha'}{16\pi G_5} \int_{\mathcal{M}} d^5x \sqrt{-g} L_{GB} \tag{3.1}$$

where,

$$L_{GB} = R_{MNPQ}R^{MNPQ} - 4R_{MN}R^{MN} + R^2. \tag{3.2}$$

The equation of motion is

$$\begin{aligned} E_{MN} = R_{MN} &- \frac{1}{2} R g_{MN} - \frac{6}{L^2} g_{MN} - \frac{\alpha'}{2} g_{MN} L_{G.B.} \\ &+ 2\alpha' \left(R_{MPQL} R_N^{\ PQL} - 2R^{PQ} R_{MPNQ} - 2R_M^{\ Q} R_{NQ} + R R_{MN}\right) = 0. \end{aligned} \tag{3.3}$$

3.2 The Counterterm and Boundary CFT Stress Tensor

As usual in gravity theories, the action (3.1) should be supplemented with suitable boundary terms, for a well-defined variational principle. For Einstein gravity, one consider the Gibbons-Hawking surface term [19]

$$I^E_{GH} = -\frac{1}{8\pi G}\int_{\partial\mathcal{M}} d^4x\sqrt{-\gamma}K \ , \tag{3.4}$$

where $\gamma_{\mu\nu}$ and K are the induced metric and the trace of the extrinsic curvature of the boundary, respectively.

$$\gamma_{\mu\nu} = g_{\mu\nu} - n_\mu n_\nu, \tag{3.5}$$

$$K_{\mu\nu} = -\frac{1}{2}\left(\nabla_\mu n_\nu + \nabla_\nu n_\mu\right) \tag{3.6}$$

and n_μ is unit outward normal vector to the asymptotic boundary hypersurface.

A similar term occurs for Gauss-Bonnet gravity and reads [20, 21]

$$I^{(GB)}_b = -\frac{1}{8\pi G}\int_{\partial\mathcal{M}} d^4x\sqrt{-\gamma}\left\{2\alpha'\left(J - 2E^{(1)}_{\mu\nu}K^{\mu\nu}\right)\right\} \ , \tag{3.7}$$

with $E^{(1)}_{\mu\nu}$ is the four-dimensional Einstein tensor of the metric $\gamma_{\mu\nu}$ and J is the trace of

$$J_{\mu\nu} = \frac{1}{3}\left(2KK_{\mu\rho}K^\rho_\nu + K_{\rho\sigma}K^{\rho\sigma}K_{\mu\nu} - 2K_{\mu\rho}K^{\rho\sigma}K_{\sigma\nu} - K^2K_{\mu\nu}\right). \tag{3.8}$$

Variation of the action $I + I^{(E)}_b + I^{(GB)}_b$ now gives an expression which does not contain normal derivatives of $\delta g_{\mu\nu}$.

It is well known that the total action has diverges even at tree level. The divergence arise from integrating over the infinite volume of spacetime. We regularise this divergence by using the procedure proposed by Balasubramanian and Kraus [17], which furnishes a method for calculating gravitational action and conserved quantities without reliance on any reference spacetime. This technique was inspired by AdS/CFT correspondence and consists of adding suitable counter terms I_{ct} to the action of the theory in order to ensure the finiteness of the boundary stress tensor [22].

We have found that the on-shell action can be regularised by the following counter term (see also [23]).

$$I_{\rm ct} = \frac{1}{8\pi G_5}\int \partial\mathcal{M} d^4x\sqrt{-\gamma}\left(c_1 + \frac{c_2}{2}\mathcal{R}\right) \tag{3.9}$$

where c_1 and c_2 are functions of α'. $\mathcal{R}$ is the Ricci scalar made out if boundary metric γ. For flat boundary geometry $(R^1 \times R^3)$, $\mathcal{R} = 0$ and the counterterm terns out to be (up to first order in α'),

$$I_{\rm ct} = \frac{1}{8\pi G_5}\int_{\partial\mathcal{M}} d^4x\sqrt{-\gamma}\left(\frac{3}{L^2} - \frac{\alpha'}{L^3}\right). \tag{3.10}$$

Varying the total action (which contains the boundary terms (3.4),(3.7) and (3.10)) with respect to the boundary metric $\gamma_{\mu\nu}$, we compute the divergence-free boundary stress-tensor

$$S_{\mu\nu} = \frac{1}{8\pi G_5}\left(K_{\mu\nu} - K\gamma_{\mu\nu} + 2\alpha'(3J_{\mu\nu} - J\gamma_{\mu\nu}) - \frac{3}{L^2}\gamma_{\mu\nu} + \frac{\alpha'}{L^3}\gamma_{\mu\nu}\right). \tag{3.11}$$

Since the CFT metric is given by,

$$h_{\mu\nu} = \lim_{\tilde{R}\to\infty} \frac{L^2}{\tilde{R}^2}\gamma_{\mu\nu}, \tag{3.12}$$

the boundary CFT stress tensor is given by [18],

$$\begin{aligned} T_{\mu\nu} &= \lim_{\tilde{R}\to\infty} \frac{\tilde{R}^2}{L^2} S_{\mu\nu} \\ &= \lim_{\tilde{R}\to\infty} \frac{\tilde{R}^2}{L^2}\frac{1}{8\pi G_5}\left(K_{\mu\nu} - K\gamma_{\mu\nu} + 2\alpha'(3J_{\mu\nu} - J\gamma_{\mu\nu}) - \frac{3}{L}\gamma_{\mu\nu} + \frac{\alpha'}{L^3}\gamma_{\mu\nu}\right) \end{aligned} \tag{3.13}$$

where $\tilde{R}$ is the cutoff in radial direction. The gauge theory lives on the boundary of the AdS space which is at $r = \tilde{R}$.

3.3 Solution to the Equation of Motions

The solution to the equation of motion is given by the metric [24],

$$ds^2 = 2dvdr - \frac{r^2}{L^2}f(br)dv^2 + \frac{r^2}{L^2}dx^i dx^i, \tag{3.14}$$

where the function $f(br)$ is given by,

$$f(br) = 1 - \frac{1}{(br)^4} + \frac{2\alpha'}{L^2}\left(1 + \frac{1}{(br)^8}\right). \tag{3.15}$$

The horizon radius r_H is given by,

$$r_H = \frac{1-\alpha'}{b} \tag{3.16}$$

and the temperature of this black brane is,

$$T = \frac{f'(r_H)}{4\pi} = \frac{1}{\pi b L^2}\left(1 - \frac{\alpha'}{L^2}\right). \tag{3.17}$$

By giving a coordinate transformation one can obtain the boosted black brane metric which is of the following form ($u_\mu = (-\gamma, \gamma\vec{\beta})$),

$$\begin{aligned} ds^2 = \quad & 2\gamma dvdr - \frac{r^2}{L^2}\gamma^2 f(br)dv^2 + \frac{r^2}{L^2}dx^i dx^i \\ & + \frac{r^2}{L^2}(\gamma^2 - 1)dv^2 - 2\gamma\beta_i dx^i dr - 2\frac{r^2}{L^2}\gamma^2(1 - f(br))\beta_i dx^i dv \\ & + \frac{r^2}{L^2}\gamma^2(1 - f(br))\beta_i\beta_j dx^i dx^j. \end{aligned} \tag{3.18}$$

The CFT metric is given by 3.12. In $\alpha' \to 0$ limit the CFT metric $h_{\mu\nu}$ is simply the Minkowski metric $\eta_{\mu\nu}$. But when we include the α' correction then the CFT metric is no longer $\eta_{\mu\nu}$. It has the following form (for the metric 3.14),

$$h_{\mu\nu} = \begin{pmatrix} -1-\frac{2\alpha'}{L^2} & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}. \tag{3.19}$$

For metric 3.18, $h_{\mu\nu}$ also has off-diagonal terms proportional to α'. If we want to keep the CFT metric to be $\eta_{\mu\nu}$ then we have to rescale either the time coordinate $v \to (1-\alpha'/L^2)\, v$ or the space coordinates $x^i \to (1+\alpha'/L^2)\, x^i$. Here we will rescale the time coordinate

$$v = \left(1-\frac{\alpha'}{L^2}\right) V. \tag{3.20}$$

In this rescaled coordinate $(V,\vec{x})$ the metric 3.14 takes the following form,

$$ds^2 = 2\left(1-\frac{\alpha'}{L^2}\right) dV dr - \frac{r^2}{L^2}\tilde{f}(br)dV^2 + \frac{r^2}{L^2}dx^i dx^i. \tag{3.21}$$

And the temperature has also been rescaled to,

$$T = \frac{1}{\pi b L^2}\left(1-\frac{2\alpha'}{L^2}\right). \tag{3.22}$$

Now we can give a coordinate transformation (boost) and the boosted black brane metric in $(V,\vec{x})$ coordinate becomes,

$$\begin{aligned} ds^2 = \quad & 2\gamma\left(1-\frac{\alpha'}{L^2}\right) dV dr - \frac{r^2}{L^2}\gamma^2\tilde{f}(br)dV^2 + \frac{r^2}{L^2}dx^i dx^i \\ &+ \frac{r^2}{L^2}(\gamma^2-1)dV^2 - 2\gamma\left(1-\frac{\alpha'}{L^2}\right)\beta_i dx^i dr - 2\frac{r^2}{L^2}\gamma^2(1-\tilde{f}(br))\beta_i dx^i dV \\ &+ \gamma^2\frac{r^2}{L^2}(1-\tilde{f}(br))\beta_i\beta_j dx^i dx^j, \end{aligned} \tag{3.23}$$

where

$$\tilde{f}(br) = \left(1-\frac{2\alpha'}{L^2}\right) f(br). \tag{3.24}$$

3.4 First Order Metric

In this section we will determine the metric to the first order in the derivative expansion. The metric and the parameters are given by Eq. 2.8, 2.9 and 2.10.

We will choose a coordinate to set $u_\mu = \{-1,0,0,0\}$ (*i.e.* $u_V = -1$, $\beta_i^{(0)} = 0$,$\gamma = 1$) and $b^{(0)} = 1$ at a given point $x^\mu = 0$[5]. Around this point the velocities and the temperature fields can be expanded up to first order in derivative,

[5]From now, our $x^\mu = \{V,\vec{x}\}$.

$$b = 1 + x^\mu \partial_\mu b^{(0)} \tag{3.25}$$

and

$$\beta_i^{(0)} = x^\mu \partial_\mu \beta_i^{(0)}. \tag{3.26}$$

So the metric up to first order in derivatives is given by,

$$\begin{aligned} ds^2 = \; & 2 \left(1 - \frac{\alpha'}{L^2}\right) dV dr - \frac{r^2}{L^2}\tilde{f}(r) dV^2 + \frac{r^2}{L^2} dx^i dx^i \\ & - 2\left(1 - \frac{\alpha'}{L^2}\right) x^\mu \partial_\mu \beta_i^{(0)} dx^i dr \\ & - 2\frac{r^2}{L^2}(1 - \tilde{f}(r)) x^\mu \partial_\mu \beta_i^{(0)} dx^i dV - \frac{r^3}{L^2} x^\mu \partial_\mu b^{(0)} \tilde{f}'(r) dV^2. \end{aligned} \tag{3.27}$$

This is the metric $g^{(0)}$ expanded up to first order in derivative.

As we explained earlier $g^{(0)}$ is not a solution to equation of motion. We have to find the fluctuation metric $g^{(1)}$, which added to $g^{(0)}$ solves the equation of motion up to first order in derivative expansion.

Because of spatial $\mathbf{SO(3)}$ symmetry of background black brane metric, we can separately solve for the $\mathbf{SO(3)}$ scalars, $\mathbf{SO(3)}$ vectors and $\mathbf{SO(3)}$ symmetric traceless components of $g^{(1)}$.

3.4.1 The Scalar Sector

The scalar components of $g^{(1)}$ are parametrised in the following way,

$$\begin{aligned} g^{(1)}_{ii}(r) &= 3\frac{r^2}{L^2} h_1(r) \quad (\textit{sum over } i) \\ g^{(1)}_{VV}(r) &= \frac{L^2}{r^2} k_1(r) \\ g^{(1)}_{Vr}(r) &= -\frac{3}{2}\left(1 - \frac{\alpha'}{L^2}\right) h_1(r). \end{aligned} \tag{3.28}$$

Here we are working in the following gauge,

$$Tr((g^{(0)})^{-1} g^{(1)}) = 0. \tag{3.29}$$

The scalar Einstein equations (equations invariant under $\mathbf{SO(3)}$ rotations) are divided up into constraint and dynamical equations.

Constraint Equation 1

The first scalar constraint is

$$\frac{r^2}{L^2}\tilde{f}(r) E_{Vr} + E_{VV} = 0, \tag{3.30}$$

which evaluates to

$$\left(1-\frac{5\alpha'}{L^2}\right)(\partial_V b^{(0)} - \frac{1}{3}\partial_i\beta_i^{(0)}) = 0$$
$$i.e. \quad \partial_V b^{(0)} - \frac{1}{3}\partial_i\beta_i^{(0)} = 0. \tag{3.31}$$

The constraint relation remains unchanged in presence of Gauss-Bonnet correction. This relation is a consequence of the conservation of boundary energy momentum tensor.

Constraint Equation 2

The second constraint equation is

$$\frac{r^2}{L^2}\tilde{f}(r)E_{rr} + E_{Vr} = 0, \tag{3.32}$$

leads to

$$12r^3h_1(r) + (3r^4-1)h_1'(r) - L^4k_1'(r) = -2L^2r^2\left(1-\frac{\alpha'}{L^2}\right)\partial_i\beta_i^{(0)}. \tag{3.33}$$

Dynamical Scalar Equation

In addition to these constraint equations we have to add one dynamical equation. Like [12] we will add the following simplest equation,

$$E_{rr} = 5h_1'(r) + rh_1''(r) = 0. \tag{3.34}$$

We will solve the equations 3.33 and 3.34 to find out the function $h_1(r)$ and $k_1(r)$. These two equations satisfy all the regularity and normalisation conditions explained in [12]. Hence the solutions are given by,

$$\begin{aligned} h_1(r) &= 0, \\ k_1(r) &= \frac{2}{3}\frac{r^2}{L^2}\left(1-\frac{\alpha'}{L^2}\right)\partial_i\beta_i^{(0)}. \end{aligned} \tag{3.35}$$

The constants appear in $h_1(r)$ and $k_1(r)$ can be set to zero following the same argument in [12]. So the scalar part of the fluctuation metric is given by [6],

$$\left(g_S^{(1)}\right)_{\alpha\beta} dx^\alpha dx^\beta = \frac{2}{3}r\left(1-\frac{\alpha'}{L^2}\right)\partial_i\beta_i^{(0)}\ dV^2. \tag{3.36}$$

[6] g_S^1, g_V^1, g_T^1 are the scalar, vector and tensor part of the fluctuation metric g^1 respectively.

3.4.2 The Vector Sector

We will parametrise the vector part of the fluctuation metric as,

$$\left(g_V^{(1)}\right)_{\alpha\beta} = 2\frac{r^2}{L^2}(1-\tilde{f}(r))j_i^{(1)}dV dx^i. \tag{3.37}$$

Constraint Equation 3

In this sector the constraint equation is given by,

$$\frac{r^2}{L^2}\tilde{f}(r)E_{ri} + E_{Vi} = 0 \tag{3.38}$$

which gives,

$$\begin{aligned} &\left(1-\frac{5\alpha'}{L^2}\right)(\partial_i b^{(0)} - \partial_V \beta_i^{(0)}) = 0 \\ &i.e.\ \ \partial_i b^{(0)} - \partial_V \beta_i^{(0)} = 0. \end{aligned} \tag{3.39}$$

Again the presence of Gauss-Bonnet term does not have any effect on this constraint equation which can be interpreted as a consequence of conservation of boundary stress tensor.

Dynamical Equation for $j_i^{(1)}(r)$

The dynamical equation in vector sector is, $E_{ri} = 0$. The equation for $j_i^{(1)}(r)$ turns out to be [7],

$$\frac{d}{dr}\left(\frac{1}{r^3}\frac{d}{dr}j_i^{(1)}(r)\right) = -3\frac{L^2}{r^2}\partial_V\beta_i^{(0)} + \frac{\alpha'}{L^2}\left(\frac{10L^2}{r^6} - \frac{3L^2}{r^2}\right)\partial_V\beta_i^{(0)}. \tag{3.40}$$

The solution is given by,

$$j_i^{(1)}(r) = L^2 r^3\left[1+\frac{\alpha'}{L^2}\left(1+\frac{2}{r^4}\right)\right]\partial_V\beta_i^{(0)}. \tag{3.41}$$

The integration constants are set to zero since the solution should be normalisable at the boundary and the stress tensor must be renormalisable [12]. So the vector part of the fluctuation metric is given by,

$$\left(g_V^{(1)}\right)_{\alpha\beta} dx^\alpha dx^\beta = 2r\partial_V\beta_i^{(0)}\left(1-\frac{\alpha'}{L^2}\right)dV dx^i. \tag{3.42}$$

[7] when $j_i^{(1)}$'s or their derivatives appear with α' then we have set the leading order (in α') values of $j_i^{(1)}$'s or their derivatives.

3.4.3 The Tensor Sector

The $\mathbf{SO(3)}$ tensor part of $g^{(1)}$ can be parametrise in the following way

$$\left(g_T^{(1)}\right)_{\alpha\beta} = \frac{r^2}{L^2}\alpha_{ij}^{(1)} dx^i dx^j \tag{3.43}$$

where $\alpha_{ij}^{(1)}$ is symmetric traceless 3×3 matrix.

Dynamical Equation for $\alpha_{ij}^{(1)}(r)$

Equation for $\alpha_{ij}^{(1)}(r)$ follows from the $E_{ij} = 0$.

$$\begin{aligned}\frac{d}{dr}\left((r^5 - r)\frac{d}{dr}\alpha_{ij}^{(1)}(r)\right) = &- 6L^2 r^2 \sigma_{ij}^{(0)} \\ &+ \frac{\alpha'}{L^2}\frac{1}{r^4}\left(2(9+5r^8)\alpha_{ij}^{'(1)}(r)\right) \\ &+ \frac{\alpha'}{L^2}\frac{1}{r^4}\left(2L^2r^2(-4+9r^4)\sigma_{ij}^{(0)} + 2r(-3+r^8)\alpha_{ij}^{''(1)}(r)\right)\end{aligned} \tag{3.44}$$

where,

$$\sigma_{ij}^{(0)} = \partial_{(i}\beta_{j)}^{(0)} - \frac{1}{3}\delta_{ij}\partial_m\beta_m^{(0)}. \tag{3.45}$$

The solution for $\alpha_{ij}^{(1)}(r)$ is given by [8],

$$\left(g_T^{(1)}\right)_{\alpha\beta} dx^\alpha dx^\beta = 2\left(r\left(1 - \frac{\alpha'}{L^2}\right) - \frac{1}{4r^2}\left(1 - \frac{8\alpha'}{L^2}\right)\right)\sigma_{ij}^{(0)} dx^i dx^j. \tag{3.46}$$

Summary of α' corrected first order calculation

The α' corrected metric $g^{(0)} + g^{(1)}$ expanded up to first order in boundary derivatives around some point x^μ where $b^{(0)} = 1$ and $u_\mu = \{-1, 0, 0, 0\}$ is given by,

$$\begin{aligned}ds^2 = &\; 2\left(1 - \frac{\alpha'}{L^2}\right) dV dr - \frac{r^2}{L^2}\tilde{f}(r) dV^2 + \frac{r^2}{L^2} dx^i dx^i \\ &- 2\left(1 - \frac{\alpha'}{L^2}\right) x^\mu \partial_\mu \beta_i^{(0)} dx^i dr \\ &- 2\frac{r^2}{L^2}(1 - \tilde{f}(r)) x^\mu \partial_\mu \beta_i^{(0)} dx^i dV - \frac{r^3}{L^2} x^\mu \partial_\mu b^{(0)} \tilde{f}'(r) dV^2 \\ &+ \frac{2}{3} r\left(1 - \frac{\alpha'}{L^2}\right)\partial_i\beta_i^{(0)}\; dV^2 + 2r\partial_V\beta_i^{(0)}\left(1 - \frac{\alpha'}{L^2}\right) dV dx^i \\ &+ 2\left(r\left(1 - \frac{\alpha'}{L^2}\right) - \frac{1}{4r^2}\left(1 - \frac{8\alpha'}{L^2}\right)\right)\sigma_{ij}^{(0)} dx^i dx^j.\end{aligned} \tag{3.47}$$

[8] In the right hand side of Eq. 3.44 we set the leading order (in α') values of $\alpha_{ij}^{(1)}$ and their derivatives.

Global solution to first order in derivative

The metric 3.47 has been calculated about $x^\mu = 0$ assuming $b^{(0)} = 1$ and $\beta_i^{(0)} = 0$. But one can also write the metric about any point. The α' corrected global metric is given by,

$$\begin{aligned}
ds^2 = \quad & -\left(1-\frac{\alpha'}{L^2}\right) u_\mu dx^\mu dr - \frac{r^2}{L^2}\tilde{f}(br)u_\mu u_\nu dx^\mu dx^\nu + \frac{r^2}{L^2}P_{\mu\nu}dx^\mu dx^\nu \\
& + 2r^2 bF(br)\sigma_{\mu\nu}dx^\mu dx^\nu + \frac{2}{3}\left(1-\frac{\alpha'}{L^2}\right) r u_\mu u_\nu \partial_\lambda u^\lambda dx^\mu dx^\nu \\
& - r\left(1-\frac{\alpha'}{L^2}\right) u^\lambda \partial_\lambda (u_\mu u_\nu) dx^\mu dx^\nu
\end{aligned} \tag{3.48}$$

where $F(r)$ is given by,

$$F(r) = \left(\frac{1}{r}\left(1-\frac{\alpha'}{L^2}\right) - \frac{1}{4r^4}\left(1-\frac{8\alpha'}{L^2}\right)\right). \tag{3.49}$$

3.5 The Energy Momentum Tensor

Once we obtained the α' corrected metric up to first order fluctuations, we can find the boundary stress tensor using the definition 3.13. Different components are given by,

$$\begin{aligned}
16\pi G_5 T_{VV} &= \frac{3}{L^5 b^4}\left(1-\frac{5\alpha'}{L^2}\right) = 3T^4\pi^4L^3\left(1+\frac{3\alpha'}{L^2}\right) \\
16\pi G_5 T_{ij} &= \frac{1}{L^5 b^4}\left(1-\frac{5\alpha'}{L^2}\right)\delta_{ij} - \frac{2}{L^3 b^3}\left(1-\frac{11\alpha'}{L^2}\right)\sigma_{ij}^{(0)} \\
&= T^4\pi^4L^3\left(1+\frac{3\alpha'}{L^2}\right)\delta_{ij} - 2T^3\pi^3L^3\left(1-\frac{5\alpha'}{L^2}\right)\sigma_{ij}^{(0)} \\
16\pi G_5 T_{Vi} &= -\frac{4}{L^5}\left(1-\frac{5\alpha'}{L^2}\right)x^\mu\partial_\mu\beta_i^{(0)} \\
&= -4T^4\pi^4L^3\left(1+\frac{3\alpha'}{L^2}\right)x^\mu\partial_\mu\beta_i^{(0)}.
\end{aligned} \tag{3.50}$$

Here we have written the energy momentum tensor about for the metric 3.47 *i.e.* about $x^\mu = 0$ assuming that $b^{(0)} = 1$ and $\beta_i^{(0)} = 0$ at the origin. But for the global metric 3.48 the energy momentum tensor can be written in a covariant way. Up to first order in derivative expansion the energy momentum tensor has the form given by Eq. 1.1 (see [26]). Therefore in presence of the Gauss-Bonnet term one can write the energy momentum tensor in a covariant form in the following way,

$$\begin{aligned}
16\pi G_5 T_{\mu\nu} &= \frac{1}{L^5 b^4}\left(1-\frac{5\alpha'}{L^2}\right)(4u_\mu u_\nu + \eta_{\mu\nu}) - \frac{2}{L^3 b^3}\left(1-\frac{11\alpha'}{L^2}\right)\sigma_{\mu\nu} \\
&= T^4\pi^4L^3\left(1+\frac{3\alpha'}{L^2}\right)(4u_\mu u_\nu + \eta_{\mu\nu}) - 2T^3\pi^3L^3\left(1-\frac{5\alpha'}{L^2}\right)\sigma_{\mu\nu}.
\end{aligned} \tag{3.51}$$

The shear viscosity coefficient is given by,

$$\eta = \frac{1}{16\pi G_5 L^3 b^3}\left(1 - \frac{11\alpha'}{L^2}\right) = \frac{T^3\pi^3 L^3}{16\pi G_5}\left(1 - \frac{5\alpha'}{L^2}\right). \tag{3.52}$$

Entropy density is given by,

$$\begin{aligned} s = \frac{S}{V_3} &= \frac{Area}{4G_5 V_3} \\ &= \frac{r_H^3}{4L^3 G_5} \\ &= \frac{1}{4L^3 b^3 G_5}\left(1 - \frac{3\alpha'}{L^2}\right) \\ &= \frac{T^3\pi^3 L^3}{4G_5}\left(1 + \frac{3\alpha'}{L^2}\right). \end{aligned} \tag{3.53}$$

where $V_3 = \int d^3x$. Hence,

$$\frac{\eta}{s} = \frac{1}{4\pi}\left(1 - \frac{8\alpha'}{L^2}\right) \tag{3.54}$$

which is in agreement with [5], [6].

4. Discussion

In this paper we have constructed the local black **D3** brane geometry in presence of Gauss-Bonnet term in the bulk action up to first order in derivative expansion. The local solution we found is non-singular except at $r = 0$. We used the counter term method of Balasubramanian and Kraus to find the boundary stress tensor , which we have expanded up to first order in derivatives, for the α' corrected metric. From the expression of stress tensor one can read the shear viscosity coefficient. We have found the α' correction to the shear viscosity coefficient and also to the viscosity entropy ratio. These results are in agreement with the existing results in the literature.

Recently in [25], authors have demonstrated that given a black brane geometry with regular event horizon the location of the horizon in radial direction turns out to be a local function of fluid dynamical variables evaluated at the corresponding points on the boundary. In presence of this regular local event horizon they constructed an appropriate area form on spatial section and then taking the pull-back of this area form to the boundary they defined a local entropy current for the dual field theory. The entropy current has the following form

$$4G_5 L^3 b^3 J_S^\mu = u^\mu + \mathcal{O}(\epsilon^2). \tag{4.1}$$

The first order ($\mathcal{O}(\epsilon)$) correction to entropy current is zero. As mentioned in [25] finding out the α' correction to entropy current would be an interesting problem to solve. Since the relations Eq. 3.31 and Eq. 3.39, which follow from the conservation of energy momentum

tensor, remain unchanged (up to an overall factor), therefore, given the α' corrected bulk metric 3.48 it is easy to check that the entropy current, up to $\mathcal{O}(\epsilon)$, has the following form ,

$$4G_5L^3b^3J_S^\mu = \left(1 - \frac{3\alpha'}{L^2}\right)u^\mu. \tag{4.2}$$

Again the entropy current does not receive any $\mathcal{O}(\epsilon)$ correction. So to find out the α' correction up to order $\mathcal{O}(\epsilon^2)$ to the entropy current, one has to first find the corrected metric up to $\mathcal{O}(\epsilon^2)$ [27].

Although we have considered only the four derivative terms in the Lagrangian but it would be very interesting to generalise this idea for any higher derivative gravity. From the string theory point of view it would be nice to construct the corrected local black **D3** brane geometry for $\mathbf{R^4}$ term [28].

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